

What Do Accelerometers Measure?

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May 8, 2005

Accelerometers measure acceleration, the time rate of change of velocity, or do they? Consider an accelerometer with its input axis horizontal under horizontal acceleration. The output would be the acceleration component along the accelerometer input axis. Now consider that the accelerometer orientation is changed so that its input axis is up. Under horizontal acceleration or no acceleration at all, the accelerometer output will be plus 1 g (i.e., $+32.2 \text{ fps}^2$). With the input axis down, the accelerometer output would be minus 1 g. These observations (and others) have led to the conclusion that an accelerometer does not measure gravity and that it actually measures the component of "total acceleration minus gravity" along its input axis. For the previous example with input axis up, the "total acceleration" along the input axis is zero, the gravitational component along the input axis is minus 1 g, and the accelerometer output from the formula is input axis "total acceleration" (zero) minus the negative 1 g gravitational component, or plus 1 g as stipulated previously. With input axis down, the correct minus 1 g accelerometer output is also obtained from the formula. In free-fall, the "total acceleration" would be the acceleration of gravity, and the accelerometer output would be zero (from the formula, total acceleration minus gravity).

But what is the accelerometer actually measuring? "Total acceleration minus gravity" is not a satisfying answer because it is based on what the instrument is not measuring. It also presumes a definition for "total acceleration" which is generally only vaguely defined if at all. Also, how does the accelerometer know what the gravity component is that it is not measuring? As an alternative, some have defined the accelerometer output as "non-gravitational" acceleration. But what is the non-gravitational acceleration that is being measured? Acceleration is generally defined as rate of change in velocity, but which velocity rate is being measured? Velocity is defined as rate of change in position, but which position is being measured? Position is generally defined as the linear displacement between two points, but which two points? To answer these questions we should first define how an accelerometer is mechanized and its principal of operation.

An accelerometer is designed to measure the force required to maintain a proof mass within the accelerometer case. For an elementary linear accelerometer, the proof mass might be connected to the case with a spring that deflects in proportion to the force applied to the proof mass. Spring deflection is sensed by a signal generator and provided as the accelerometer output. A more sophisticated accelerometer implementation might use a detector that senses movement of the proof mass from a nominal position in the case. The detector provides input to an electrical circuit that generates a control signal to a transducer. The transducer generates force on the proof mass proportional to the transducer input, thereby maintaining the proof mass at the nominal case position. For this so-called "force-rebalance" approach, the transducer input signal is proportional to the force applied to the proof mass and is also provided as the accelerometer output.

The principal of operation for the accelerometer is based on Newton's second law of motion; the time rate of change of a body's velocity (its acceleration) is proportional to the force applied to the body. The proportionality factor is the reciprocal of the body's mass. Since the accelerometer is mechanized to output a signal proportional to force on its proof mass, the accelerometer output thereby becomes proportional to the rate of change of proof mass velocity (i.e., proof mass acceleration). Because the proof mass is constrained by the force transducer to maintain its position in the accelerometer case, the proof mass acceleration equals the case acceleration. Therefore, the accelerometer output becomes proportional to the acceleration of the accelerometer case (or the base upon which the accelerometer is mounted). To generate an equivalent acceleration signal, the accelerometer signal generator output is normalized by its proof mass magnitude and force transducer scale factor. The normalized output is sometimes denoted as "specific force".

But what exactly does specific force measure? To directly address this question let us represent the accelerometer as a generalized free mass that can measure and output its specific force. Consider two such free masses in the same uniform gravity field; mass 1 and mass 2. In the absence of applied forces, the relative velocity (time rate of change of distance) between the two masses will remain constant (i.e., both will be in "free-fall" and accelerating at the same gravitational value). This is a generalization of Newton's first law of motion to include the effect of uniform gravity and to define relative velocity as being the rate of change of displacement between two free masses in the same gravity field. Now consider that a force is applied only to mass 2. The relative velocity between masses 1 and 2 will then increase at a rate (acceleration) equal to the applied force divided by the mass of mass 2. The mass 2 specific force output will exactly measure this acceleration. This is a generalization of Newton's second law of motion to include the effect of gravity and to identify the acceleration response of the proof mass as being relative to a non-forced reference (mass 1) contained in the same gravity field.

What if we now stipulate that masses 1 and 2 are in different gravity fields 1 and 2. If no forces are applied to either mass, the relative velocity between the two masses will have a time rate of change equal to the difference between the values of gravity field 1 and gravity field 2. Under this condition, the specific force measurements for masses 1 and 2 will each be zero. If we now apply forces to the masses, an additional relative acceleration will be created between the two equal to the difference between the specific force measured by masses 1 and 2 (i.e., exactly as described in the previous paragraph). It is also to be noted based on the above discussion that the relative difference between gravity fields 1 and 2 can be determined by measuring the relative acceleration between masses 1 and 2 (e.g., by optical means) and subtracting the difference between the mass 1 and 2 specific force measurements. It is stated without proof that the absolute value of a gravity field is immeasurable.

Based on the above discussion, we can postulate the answer to our original question; what do accelerometers measure? Accelerometers measure specific force, the time rate of change of velocity relative to local gravitational space. Local gravitational space is then defined as the same space occupied by the accelerometer but which contains a virtual mass with zero applied specific force (analogous to the generalized case described previously in which mass 2 with applied force represents the accelerometer, mass 1 without applied force represents the virtual mass, and both masses 1 and 2 are in the same

gravitational field). Specific force is a basic absolute quantity that is measured directly by the accelerometer. If the accelerometer is mounted to a vehicle that is exposed to force, the accelerometer output will measure the specific force response of the vehicle (which is the same as that of the accelerometer attached to the vehicle). This response will be identical for the same applied specific force no matter what value of local gravity surrounds the vehicle. Gravity is a property of space whose value at a particular location can only be determined relative to the value of gravity at another location.

Using the above observations, let us now define some basic equations relating position, velocity, acceleration and specific force between two general locations 1 and 2:

$$\begin{aligned} \underline{v}_{2/1} &\equiv \frac{d}{dt} \underline{R}_{2/1} \\ \underline{a}_{2/1} &\equiv \frac{d}{dt} \underline{v}_{2/1} \\ \underline{a}_{2/1} &= \underline{a}_{SF2} + \underline{g}_2 - \underline{a}_{SF1} - \underline{g}_1 \end{aligned} \quad (1)$$

where

- $\underline{R}_{2/1}$ = Linear displacement vector of point 2 relative to point 1.
- $\underline{v}_{2/1}$ = Velocity vector of point 2 relative to point 1.
- $\underline{a}_{2/1}$ = Acceleration vector of point 2 relative to point 1.
- $\underline{a}_{SF1}, \underline{a}_{SF2}$ = Total specific force vector being applied at points 1 and 2 (measurable by accelerometers).
- $\underline{g}_1, \underline{g}_2$ = Gravity vector at points 1 and 2.

Accelerometer triads (three-orthogonal accelerometers) at locations 1 and 2 would directly measure the components of \underline{a}_{SF1} and \underline{a}_{SF2} .

The above equations satisfy the case described previously in which points 1 and 2 represent free masses in the same uniform gravity field and only mass 2 is exposed to force. Then $\underline{g}_2 = \underline{g}_1$, $\underline{a}_{SF1} = 0$ and $\frac{d}{dt} \underline{v}_{2/1} = \underline{a}_{SF2}$. Thus, an accelerometer triad at location 2 would measure proof mass acceleration relative to mass 1 (the local gravitational space reference).

As a more familiar case, consider an accelerometer triad in an inertial navigation system (INS) used to calculate position relative to earth's center. For this case, consider point 1 as being the center of the earth and point 2 as being in the INS. For simplicity, assume that the earth is not rotating. Both the earth and the INS are in the gravity field generated by the universe which we can approximate as being the same at earth's center and at the INS. At the INS location there is an additional gravity field created by the earth itself.

Thus, $\underline{g}_1 \approx \underline{g}_{\text{Universe}}$, $\underline{g}_2 \approx \underline{g}_{\text{Universe}} + \Delta \underline{g}_{2\text{Earth}}$, $\underline{g}_{\text{Universe}}$ is the gravity field created by the universe (exclusive of the earth) and $\Delta \underline{g}_{2\text{Earth}}$ is the additional gravity (relative to gravity at earth's center) created by the earth at the INS. We can approximate the total force on the earth as zero so that $\underline{a}_{SF1} \approx 0$. Then the previous equations simplify to:

$$\begin{aligned}\frac{d}{dt} \underline{v}_{2/1} &= \underline{a}_{SF_2} + \Delta \underline{g}_{2\text{Earth}} \\ \frac{d}{dt} \underline{R}_{2/1} &= \underline{v}_{2/1}\end{aligned}\tag{2}$$

a basic form familiar to most inertial navigation engineers. Integrating the above equations in the INS computer using \underline{a}_{SF_2} measured by the INS accelerometers, provides velocity and position relative to earth's center. The $\Delta \underline{g}_{2\text{Earth}}$ term in Equations (2) would be modeled in the INS computer based on previously determined values of gravity as a function of position relative to earth's center.

What if we want the INS (point 2) to measure motion relative to another point 3 fixed to the earth. From Equations (1) we can directly write

$$\begin{aligned}\frac{d}{dt} \underline{v}_{2/3} &= \underline{a}_{SF_2} - \underline{a}_{SF_3} + \underline{g}_2 - \underline{g}_3 \\ \frac{d}{dt} \underline{R}_{2/3} &= \underline{v}_{2/3}\end{aligned}\tag{3}$$

A difficulty with Equations (3) is that the specific force \underline{a}_{SF_3} at point 3 must be previously determined and included in the INS computer memory. In addition, gravity at point 2 must be modeled relative to gravity at point 3. Available gravity models are always referenced to earth's center, previously point 1. By selecting the reference point as the center of the earth where the specific force \underline{a}_{SF_1} is approximately zero, both difficulties are avoided. An alternative approach is to note that the relative acceleration \underline{a}_{3-1} between earth fixed points 1 and 3 is zero for which Equations (1) show that $\underline{a}_{SF_3} + \underline{g}_3 = \underline{a}_{SF_1} + \underline{g}_1 \approx \underline{g}_1$. Substituting in Equations (3) then gives

$$\begin{aligned}\frac{d}{dt} \underline{v}_{2/3} &= \underline{a}_{SF_2} + \underline{g}_2 - \underline{g}_1 = \underline{a}_{SF_2} + \Delta \underline{g}_{2\text{Earth}} \\ \frac{d}{dt} \underline{R}_{2/3} &= \underline{v}_{2/3}\end{aligned}\tag{4}$$

which uses a gravity model referenced to earth's center as in Equations (2), but which references the INS position and velocity to point 3 rather than point 1. The identical result could have been obtained more easily by noting that $\underline{R}_{2/3} = \underline{R}_{2/1} - \underline{R}_{1/3}$, and because $\underline{R}_{1/3}$ is constant (i.e., points 1 and 3 are fixed to the earth),

$\underline{v}_{2/3} \equiv \frac{d}{dt} \underline{R}_{2/3} = \frac{d}{dt} \underline{R}_{2/1} = \underline{v}_{2/1}$. Substitution in Equations (2) then provides Equations (4) directly.

Now consider mass 2 to be in free-fall whose position relative to earth's center is to be calculated (e.g., a satellite). This is identical to the previous INS cases except the specific

force \underline{a}_{SF_2} in the satellite would now be zero. It also implicitly shows how a free-fall satellite environment can be created artificially; by producing an environment in which \underline{a}_{SF_2} is zero. This can be achieved in an airplane for example, by flying a trajectory in which lift, thrust and side force are controlled so that accelerometers mounted in the aircraft (that measure \underline{a}_{SF_2}) have an average reading of zero.

Finally, consider the earth and its oceans. Consider two distant locations at the earth surface that are fixed relative to the earth. Because these points are defined to be fixed, the relative acceleration between them ($\underline{a}_{2/1}$) will be zero. Now consider that the gravity fields at points 1 and 2 have rates of change so that over time, the difference in gravity values ($\underline{g}_2 - \underline{g}_1$) changes (this is caused principally by the gravity field created by moon's mass as it changes its position relative to the earth due to earth's rotation and moon's orbit). From the derivative of the third of Equations (1) we see that $\underline{a}_{SF_2} - \underline{a}_{SF_1}$ (the difference in specific force between points 1 and 2) will have a rate of change equal to the rate of change in $\underline{g}_2 - \underline{g}_1$. The changing specific force between points 1 and 2 is felt by the oceans at points 1 and 2, thereby producing relative movement of the ocean waters between points 1 and 2. The result is the tidal flows generated on the earth's surface.